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Environmental Implementation Guide for Radiological Survey Procedures

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APPENDIX B

DERIVATION OF ALPHA SCANNING
EQUATIONS

Probability of Detecting Surface Contamination while Surveying for Alpha-Emitting Radionuclides

For alpha survey instrumentation with a background around 1 to 3 counts per minute (cpm), a single count will give a surveyor sufficient cause to stop and investigate further. Assuming this to be true, the probability of detecting given levels of alpha-emitting radionuclides can be calculated by use of Poisson summation statistics.

Experiments yielding numerical values of a random variable x where x represents the number of outcomes occurring during a given time interval or a specified region in space are often called Poisson experiments. The probability distribution of the Poisson random variable x, representing the number of outcomes occurring in a given time interval *t*, is given by the following:

$$P(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!},\ x = 0,1,2,\ldots \tag{B.1}$$

where

$P(\times;\lambda t)$ = probability of $\times$ number outcomes in time interval *t*,
λ = average number of outcomes per unit time,
λt = average value expected.

To define this distribution for an alpha scanning system, substitutions may be made giving the following equation:

$$P(n;m) = \frac{e^{-m}m^n}{n!} \tag{B.2}$$

where

$P(n;m)$ = probability of getting *n* counts when the average number expected is *m*,
m = λt average number of counts expected,
n = $\times$ average number of counts detected.

For a given detector size, source activity, and scanning rate, the probability of getting *n* counts while passing over the source activity with the detector can be written as follows:

$$P(n;m) = \frac{e^{-\frac{GEd}{60v}}\left[\frac{GEd}{60v}\right]^n}{n!} = \frac{e^{-\frac{GEt}{60}}\left[\frac{GEt}{60}\right]^n}{n!} \tag{B.3}$$

where

G = source activity (dpm)
E = detector efficiency (4π),
d = width of the detector in the direction of scan (cm),
t = d/v = dwell time over source (*s*),
v = scan speed (cm / *s*).

If we assume no background counts while passing over the source area, then the probability of observing greater than or equal to 1 count, $P(n \geq 1)$, within a time interval t is this:

$$P(\geq 1) = 1 - \sum_{i=0}^{n-1} P(i, m), i = 0,1,2,\ldots \tag{B.4}$$

If we assume further that a single count is sufficient to cause a surveyor to stop and investigate further then the following applies:

$$P(\geq 1) = 1 - P(n = 0) = 1 - e^{\frac{-GEd}{60\,v}} \tag{B.5}$$

Figures B.1 through B.4 show this function plotted for three different detector sizes and four different source activity levels. Note that the source activity levels are given in terms of absolute activity values (dpm), the probe sizes are the dimensions in the direction of scanning, and the detection efficiency has been assumed to be 15%. If the assumption is made that the areal activity is contained within a 100-cm^2 area and that the detector completely passes over the area either in one or multiple passes, then the activity levels can be stated in areal units (dpm / 100 cm^2).

Once a count has been recorded and the surveyor stops, the surveyor should wait a sufficient period of time such that if the guideline level of contamination is present, then the probability of getting another count is at least 90 %. This minimum time interval can be calculated for given contamination guideline values by substituting the following parameters into Eq. (B.5) and solving as follows:

$P(\geq 1) = 0.9$
$d/v = t$
$G = CA/100$ where C = contamination guideline (dpm / 100 cm^2)
A = detector area (cm^2)

giving

$$t = \frac{13800}{CAE} \tag{B.6}$$

Equation (B.3) can be solved to give the probability of getting any number of counts while passing over the source area, although the solutions can become long and complex. Many portable proportional counters have background count rates on the order of 5 to 10 cpm and a single count will not give a surveyor cause to stop and investigate further. If a surveyor did stop for every count, and subsequently waited a sufficiently long period to make sure that the previous count either was or was not caused by an elevated contamination level, then little or no progress would be made. For these types of instruments, the surveyor usually will need to get at least 2 counts while passing over the source area before stopping for further investigation. Assuming this to be a valid assumption, Eq. (B.3) can be solved for $n = 2$ giving the following:

$$P(n \geq 2) = 1 - P(n = 0) - P(n = 1) \tag{B.7}$$

$$= 1 - e^{-\frac{(GE+B)\,t}{60}} - \frac{(GE+B)t}{60}\, e^{-\frac{(GE+B)\,t}{60}}$$

$$= 1 - e^{-\frac{(GE+B)\,t}{60}} \left(1 + \frac{(GE+B)t}{60}\right)$$

where

$P(n \geq 2)$ = probability of getting 2 or more counts during the time interval t,
$P(n = 20)$ = probability of not getting any counts during the time interval t,
$P(n = 1)$ = probability of getting 1 count during the time interval t,
B = background count rate (cpm).

All other variables are the same as in Eq. (B.3).

Figures 5 and 6 show this function plotted for three different probe sizes and two different source activity levels. The same assumptions were made when calculating these curves as were made for Figs. 1 through 4 except that the background was assumed to be 7 cpm.

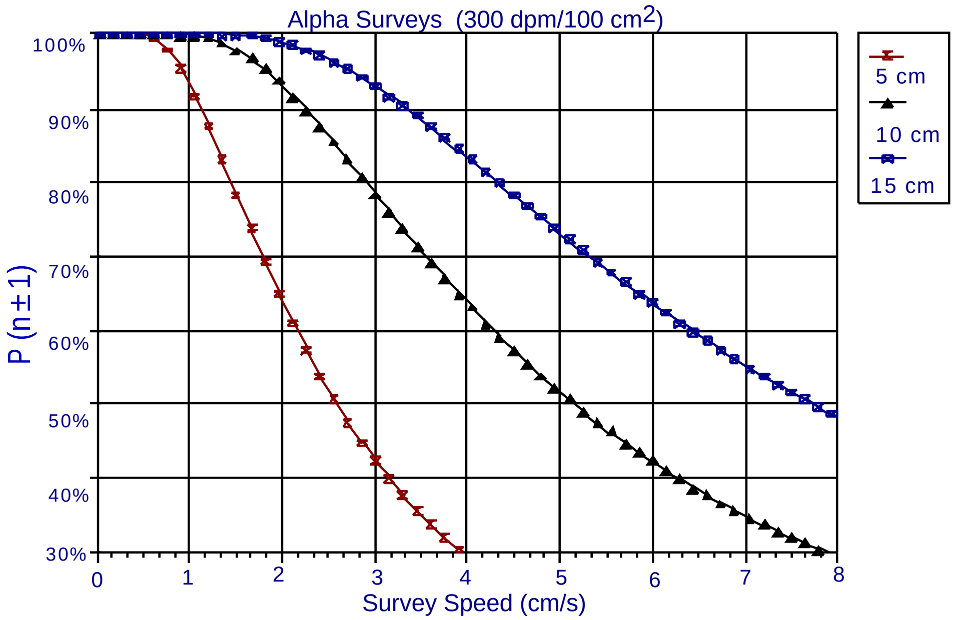

Fig. B.1. Probability of detecting an alpha radiation activity level of 5000 dpm at survey speeds of 0 to 20 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

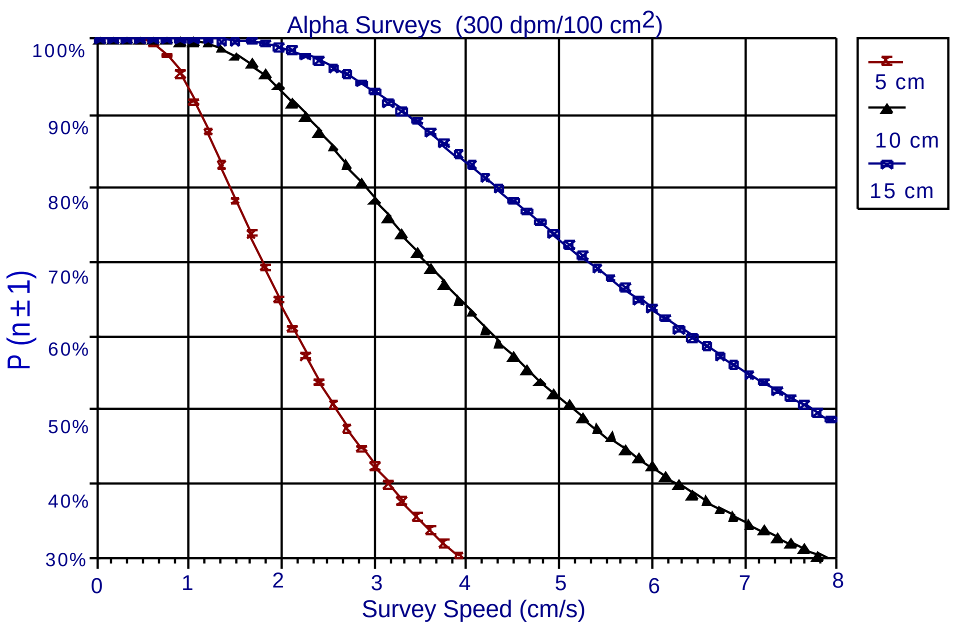

Fig. B.2. Probability of detecting an alpha radiation activity level of 1000 dpm/100 cm^2 at survey speeds of 0 to 40 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

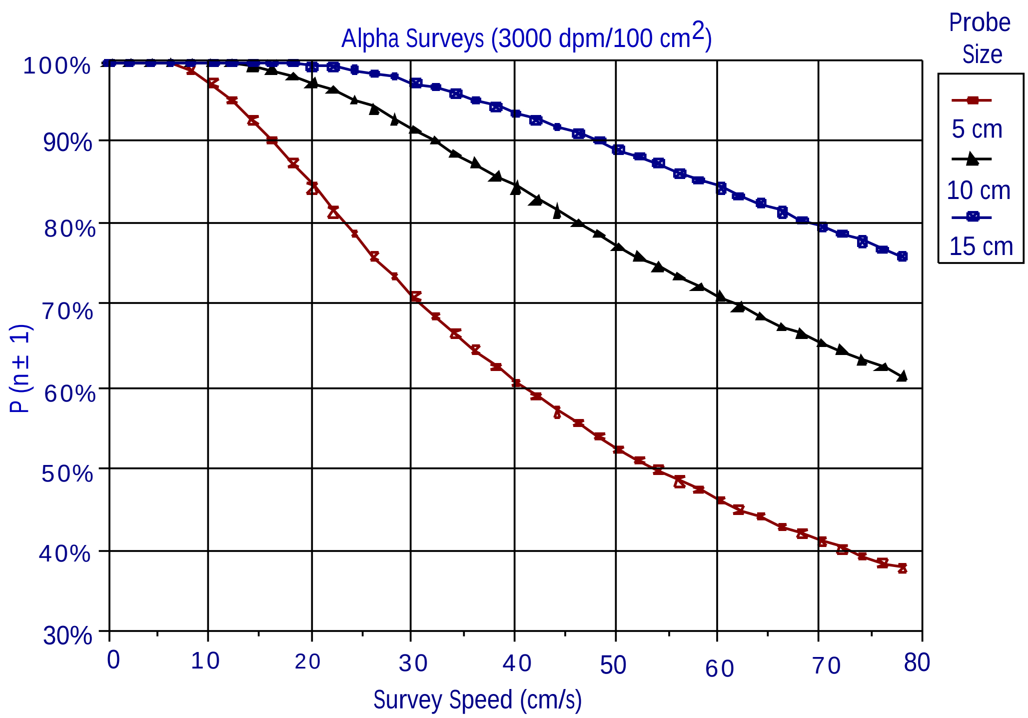

Fig. B.3. Probability of detecting an alpha radiation activity level of 3000 dpm/100 cm² at survey speeds of 0 to 80 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

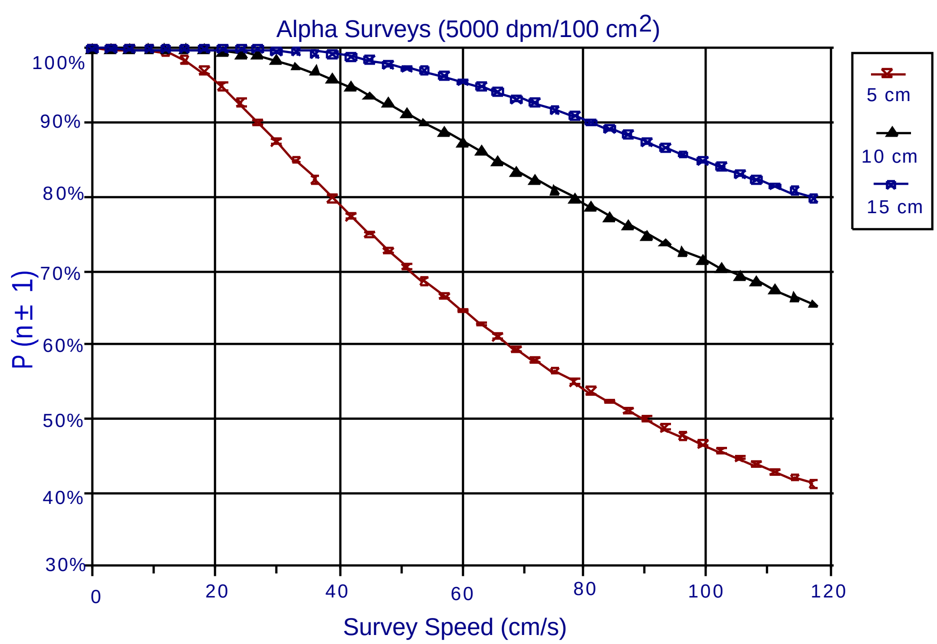

Fig. B.4. Probability of detecting an alpha radiation activity level of 5000 dpm/100 cm² at survey speeds of 0 to 120 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

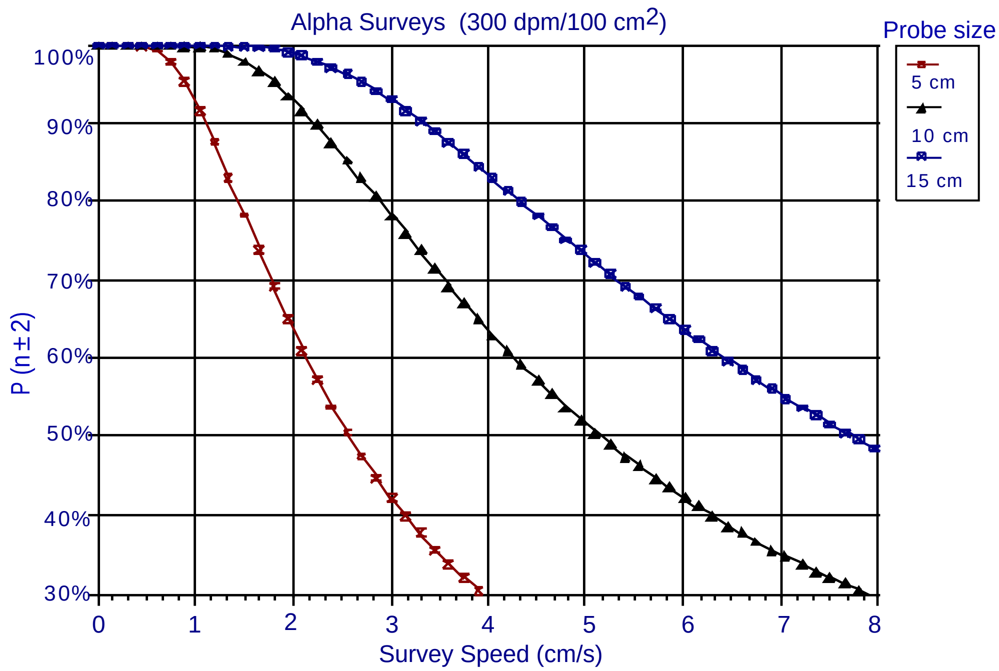

Fig. B.5. Probability of detecting an alpha radiation activity level of 300 dpm/100 cm² at survey speeds of 0 to 8 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

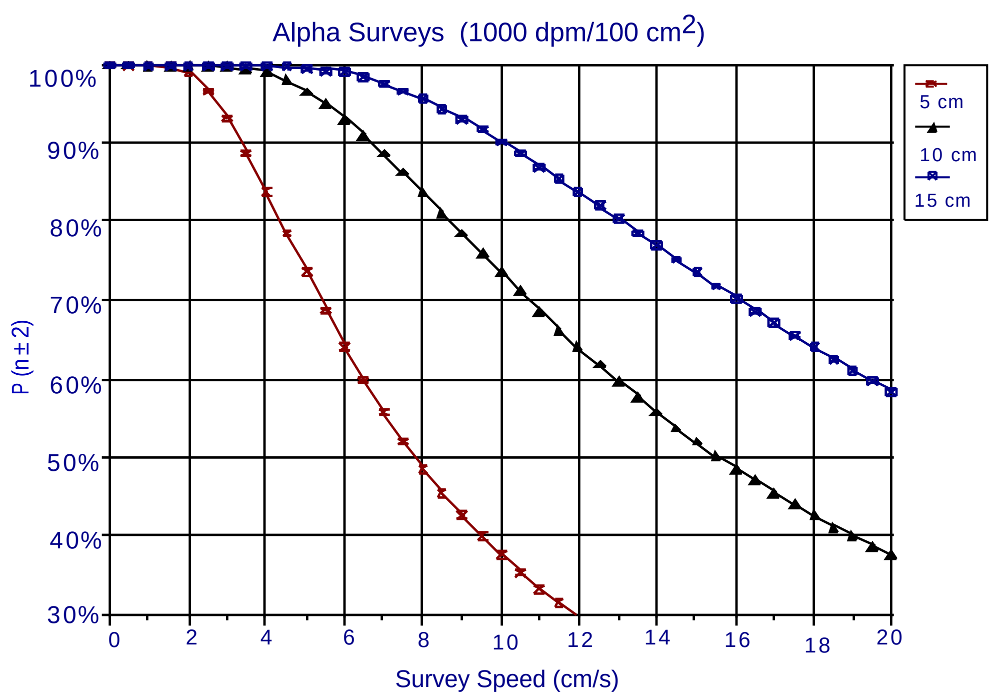

Fig. B.6. Probability of detecting an alpha radiation activity level of 1000 dpm/100 cm² at survey speeds of 0 to 20 cm/s and at probe diameters of 5-, 10-, and 15-cm (Sect. 5).

APPENDIX C

NON-PARAMETRIC TEST WHEN THE BACKGROUND
VALUE IS NOT PRECISELY KNOWN

NON-PARAMETRIC TEST WHEN BACKGROUND IS NOT PRECISELY KNOWN

Tests when the background is not precisely known.

The tests in Sects. 7.6.2 and 7.6.3 are based on the assumption that "background" is a known constant value that is subtracted from each measurement before the test is conducted. These tests are not appropriate when the background value has uncertainty. In this section, two tests are illustrated that are appropriate when the background mean is not known with certainty. The first test, which is appropriate when the data are normally distributed, is a modification of the test in Sect. 7.6.2 that uses Eq. (7.13). The second test is a nonparametric procedure that can be used for any data distribution. This latter test is preferred over the normal theory test unless the normality assumption is clearly appropriate.

Tests based on normal distribution assumption

In this section a modified version of the test in Sect. 7.6.2 is presented that may be used when the background value is a mean computed using n_b background measurements collected at random from a suitable background area during a suitable time period. This test should be used only when the data are known with confidence to be normally distributed.

The upper 95% confidence limit on the true mean for the survey unit is computed using the following equation instead of Eq. (7.13).

$$\mu_{\alpha,b} = \bar{x} + t_{0.95,df}\, s_{x\mathrm{bar}} \tag{7.13b}$$

where

$\mu_{\alpha,b}$ = estimated upper 95% confidence limit on the true background-corrected mean for the survey unit

$\bar{x}$ = mean of the n_s background-corrected survey-unit measurements

= mean of survey-unit measurements - mean of background measurements

$s_{x\mathrm{bar}} = (v_s + v_b)^{1/2}$

$v_s = s_s^2/n_s$

$v_b = s_b^2/n_b$

s_s^2 = estimated variance of the survey-unit measurements (before background is subtracted) computed using Eq. (7.12),

s_b^2 = estimated variance of the background measurements computed using Eq. (7.12),

n_s = number of survey-unit measurements,

n_b = number of background measurements, and

$$df = \frac{(v_s + v_b)^2}{v_s^2/(n_s - 1) + v_b^2/(n_b - 1)}$$

Source: Snedecor and Cochran, p. 97, 1980.

This formula for df is appropriate when the variance of the n_s survey-unit measurements (computed before background is subtracted) does not equal the variance of the n_b background measurements. If the two variances are equal, then $df = n_s + n_b - 2$. This latter formula for df is not recommended unless variances computed on the basis of 20 or more measurements in both the survey unit and the background area indicate that it is reasonable to assume equal variances.

Example 3

Suppose the following $n_s = 10$ values represent the activity within 10 systematic grid blocks across the survey unit being evaluated:

7.8	8.9
15	8.3
2.3	2.5
4.5	3.9
4.7	4.0

For these data: mean = 6.19,

s_s^2 = 15.0966, and

v_s = 1.50966.

Also, suppose the following $n_b = 5$ background measurements (pCi/g) have ben taken at 5 random soil sampling locations in a suitable background area:

1.5	0.9
2.3	1.4
0.7	

For these data:

background mean = 1.36

s_b^2 = 0.388 and,

v_b = 0.0776.

Therefore,

$$\bar{x} = 6.19 - 1.36 = 4.83$$

$$s_{xbar} = (1.50966 + 0.0776)^{1/2} = 1.25986$$

and

$$df = \frac{(1.50966+0.0776)^2}{2.27904/9+0.00602/4}$$

$$= 9.89$$

which is rounded down to 9. We find from Table 7.2 that

$$t_{0.95,9} = 1.833.$$

Therefore, computing Eq. (7.13b):

$$\mu_{\alpha,b} = 4.93 + 1.833(1.25986)$$
$$= 7.14$$

Suppose the guideline value is 5 units above background. In that case, the survey unit does not meet the guideline value because 7.14 >5.

Nonparametric test

In the preceding section, the test for compliance was conducted by comparing the upper 95% confidence limit on the true background-corrected mean for the survey unit [Eq. (7.13b)] with the guideline limit. That test requires the data to be normally distributed. In this section, a nonparametric (distribution-free) upper 95% confidence limit on the parameter Δ is compared with the guideline value, where Δ is the amount that survey-unit measurements exceed background, on the average. This latter method can be used regardless of the type of data distribution.

The test procedure is as follows (an example is given below):

Step 1. Compute all $n_s n_b$ differences between the n_s survey-unit measurements and the n_b background measurements. That is, compute the $n_s n_b$ differences.

$$x_{ji} = z_j - y_i$$

where

z_j = the jth survey unit measurement (not corrected for background)
y_i = the ith background measurement

Step 2. Order (rank) the $n_s n_b$ differences (x_{ji}) from smallest to largest. A computer can be programmed to compute and rank the x_{ji}.

Step 3. Compute the quantity C,

$$C = n_s n_b / 2 - 1.645[n_s n_b(n_s + n_b + 1) / 12]^{1/2}$$

and round this value to the nearest integer. [Note: the value 1.645 in this equation will change if the confidence required in the decision is different than

95%. The required constant is obtained from the standard normal distribution table found in; e.g., Gilbert (1987, Table A.1).]

If both n_s and n_b are not greater than 5, then the above formula for C should not be used. Instead, compute C using the table look-up and computation procedure described in Hollander and Wolfe (1973, pp. 78-79).

Step 4. Compute the quantity $n_s n_b + 1 - C$.

Step 5. Determine the upper 95% confidence limit on Δ. This upper limit is the $(n_s n_b + 1 - C)$th largest of the $n_s n_b$ differences, counting from the smallest x_{ji} measurement. Denote this confidence limit by $\mu_{\alpha,np}$.

Step 6. If $\mu_{\alpha,np}$ is less than the guideline value, then the survey unit being tested meets the guideline at the 95% confidence level.

<u>Example of nonparametric test</u>

The data used in the preceding example are used here. There are

n_s = 10 survey-unit measurements plotted, and
n_b = 5 background measurements, yielding
x_{ji} = 50 differences.

Step 1. The 50 differences (x_{ji}) are shown in the following table (e.g., 7.8 – 1.5 = 6.3 is the first entry).

	Survey-unit measurements									
Background measurements	7.8	15	2.3	4.5	4.7	8.9	8.3	2.5	3.9	4.0
1.5	6.3	13.5	0.8	3.0	3.2	7.4	6.8	1.0	2.4	2.5
2.3	5.5	12.7	0.0	2.2	2.4	6.6	6.0	0.2	1.6	1.7
0.7	7.1	14.3	1.6	3.8	4.0	8.2	7.6	1.8	3.2	3.3
0.9	6.9	14.1	1.4	3.6	3.8	8.0	7.4	1.6	3.0	3.1
1.4	6.4	13.6	0.9	3.1	3.3	7.5	6.9	1.1	2.5	2.6

Step 2. Listing the x_{ji} values (from the table in Step 1) from smallest to largest gives:

x_{ji}	Rank	x_{ji}	Rank	x_{ji}	Rank	x_{ji}	Rank	x_{ji}	Rank
0.0	1	1.7	11	3.1	21	5.5	31	7.4	41
0.2	2	1.8	12	3.1	22	6.0	32	7.5	42
0.8	3	2.2	13	3.2	23	6.3	33	7.6	43
0.9	4	2.4	14	3.2	24	6.4	34	8.0	44
1.0	5	2.4	15	3.3	25	6.6	35	8.2	45
1.1	6	2.5	16	3.3	26	6.8	36	12.7	46
1.4	7	2.5	17	3.6	27	6.9	37	13.5	47
1.6	8	2.6	18	3.8	28	6.9	38	13.6	48
1.6	9	3.0	19	3.8	29	7.1	39	14.1	49
1.6	10	3.0	20	4.0	30	7.4	40	14.3	50

Step 3. As both n_s and n_b are greater than or equal to 5, C is determined as follows:

$$\begin{aligned} C &= 10 \bullet 5/2 - 1.645\,(10 \bullet 5 \bullet 16/12)^{1/2} \\ &= 11.57 \end{aligned}$$

which is rounded to 12.

Step 4. $n_s\, n_b + 1 - C = 51 - 12 = 39$.

Step 5. From Step 4, the upper 95% confidence limit on Δ, $\mu_{a,np}$, is the 39th largest value of x_{ji}, which is 7.1 (from the table in Step 2).

Step 6. Compare $\mu_{\alpha,np}$ to the guideline value. From Step 5, $\mu_{\alpha,np} = 7.1$. Suppose the guideline value is 5 units above background. In that case, the survey unit does not meet the guideline value because 7.1 >5.

For example, the nonparametric 95% upper confidence limit on Δ (7.1) is almost identical to the 95% upper confidence limit on the background-corrected mean (7.14) obtained in the previous example. Hence, both tests indicated the survey unit does not meet the guideline value of 5. However, both tests will not always give the same conclusion. Preference should be given to results obtained using the nonparametric limit ($\mu_{\alpha,np}$) because it does not require the data to be normally distributed. Among the four tests described in Sects. 7.6.2 and 7.6.3 the test based on $\mu_{\alpha,np}$ is the most generally applicable because it takes into account variability among both the background and survey-unit measurements and it does not require the data to be normally distributed.

Note that an easily computed estimate of Δ (the amount that survey-unit measurements exceed background on the average) is the sample median of the $n_s n_b$ values of x_{ji}. If $n_s n_b$ is an even number, then the sample median is the arithmetic mean of the ($n_s n_b / 2$)th and the [($n_s n_b / 2$) +1]th largest values of x_{ji}. If $n_s n_b$ is an odd number, then the sample median is just the [($n_s n_b / 2$) + 1]th largest value. In the example above, $n_s n_b = 50$. Hence, the sample median is the arithmetic mean of the 25th and 26th largest values of x_{ji}, or 3.3 See Hollander and Wolfe (1973, pp. 75-78) for further discussion.